
Spin Distributions in the Pre-Equilibrium Process

T. Kawano, M.B. Chadwick

T-16, Los Alamos National Laboratory
Los Alamos, New Mexico 87545, USA

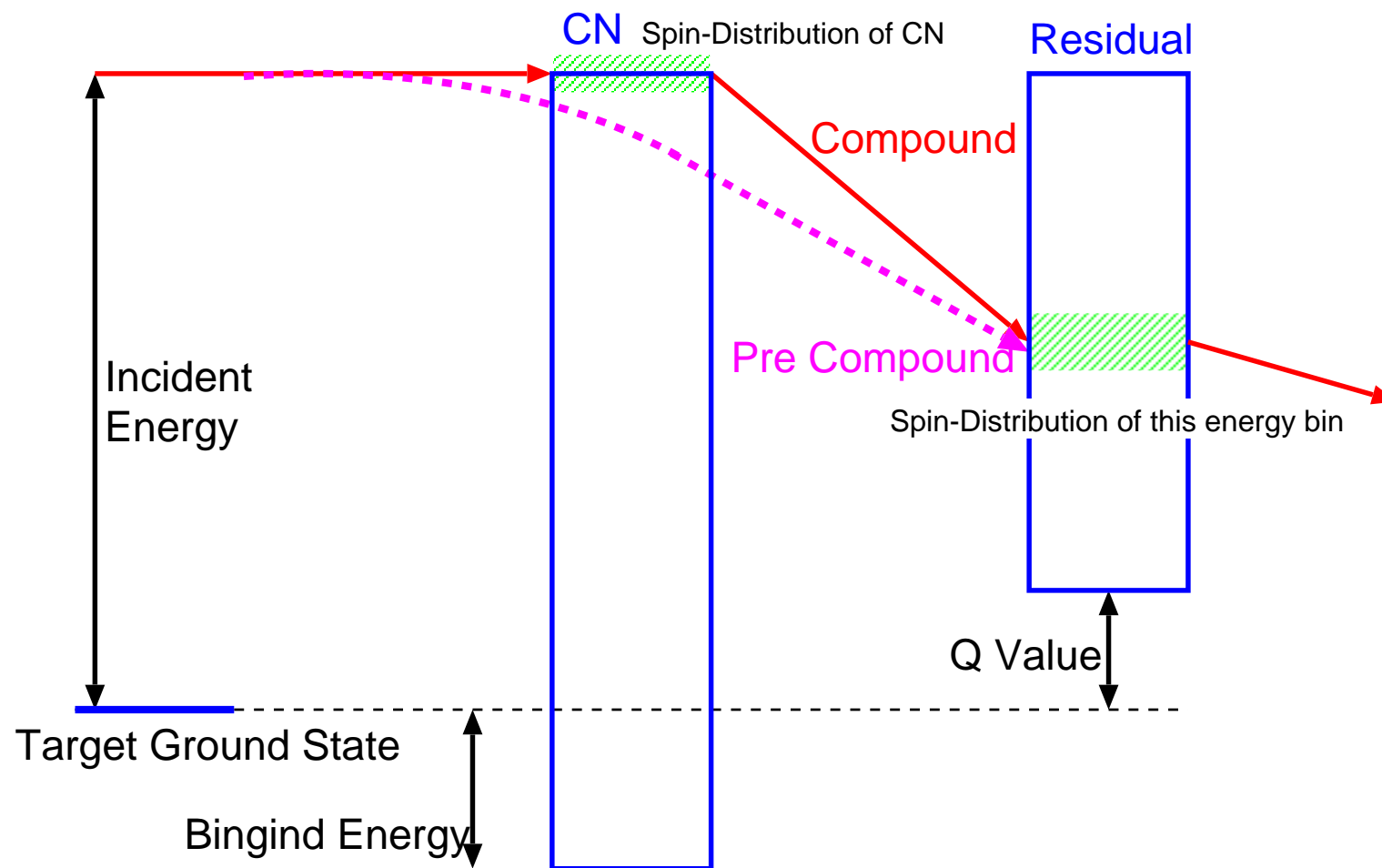
Compound Nuclear Reaction

- Spin Distribution of Compound Nucleus
 - $n + A \rightarrow (A + 1)^*$
 - Spin distribution of $(A + 1)^*$ is calculated with the optical model transmission coefficients, T_j .

$$R(J) \quad |I - j| \leq J \leq I + j$$

- Spin Distribution of the Residual Compound Nucleus (Decay of CN)
 - $(A + 1)^* \rightarrow n' + A^*$
 - Spin distribution of A^* is calculated from:
 - the first compound nucleus, $R(J)$,
 - transmission of emitted particle, $T'(j)$, and
 - spin distribution of residual nucleus (continuum), $R'(E_x, J)$.

Compound or Pre-compound Process

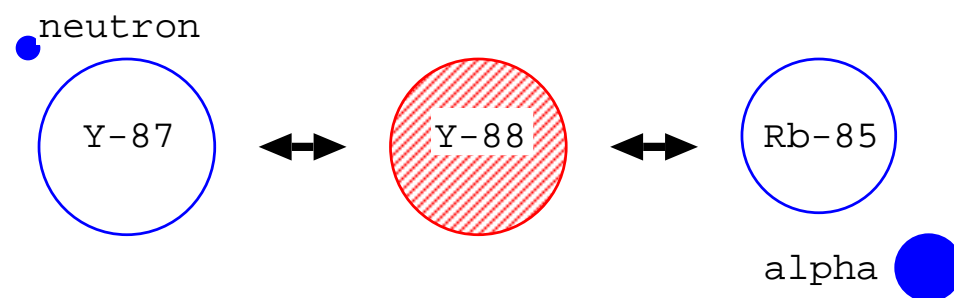


Compound Formation Cross Section

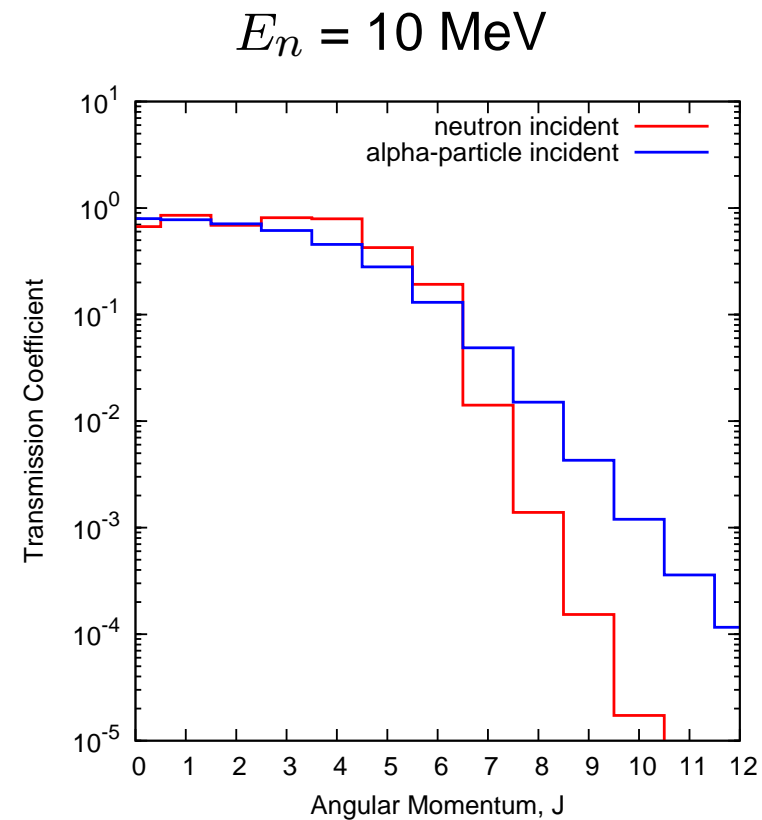
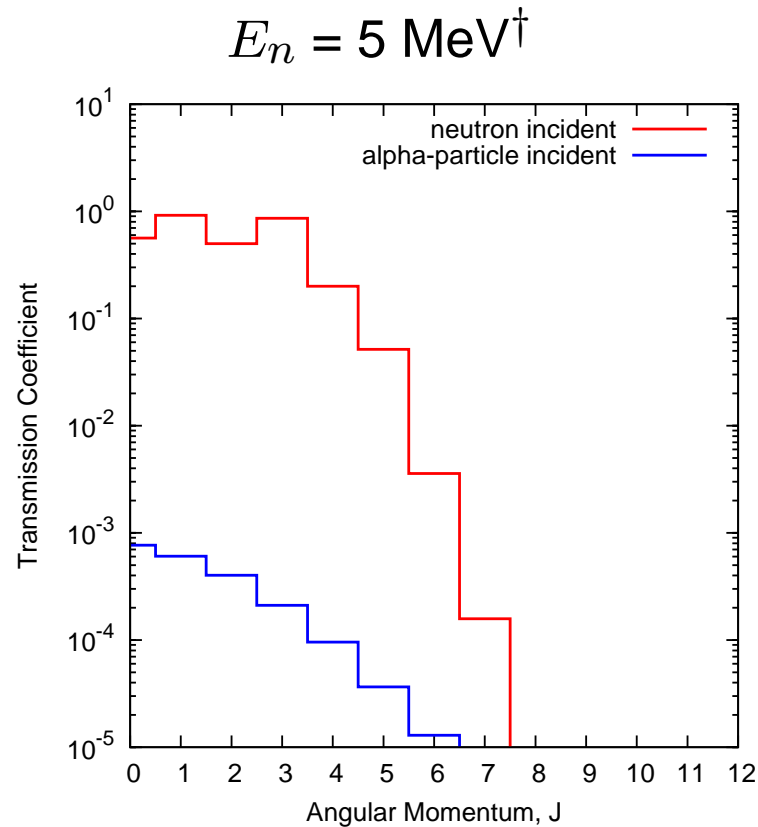
- Decay of CN is independent of the formation of CN,
- however spin distribution of initial CN population depends on the transmission of incident particle.

Surrogate Reaction (example)

- For the neutron incident reactions on ^{87}Y , the compound state is ^{88}Y .
- The same compound can be formed by α -particle incident reactions on ^{85}Rb .



Transmission (Optical Model Calc.)



† Equivalent neutron energy,

$$E_n^{CMS} = E_\alpha^{CMS} - Q$$

■ Pre-Equilibrium Theories (I)

Classical Theory

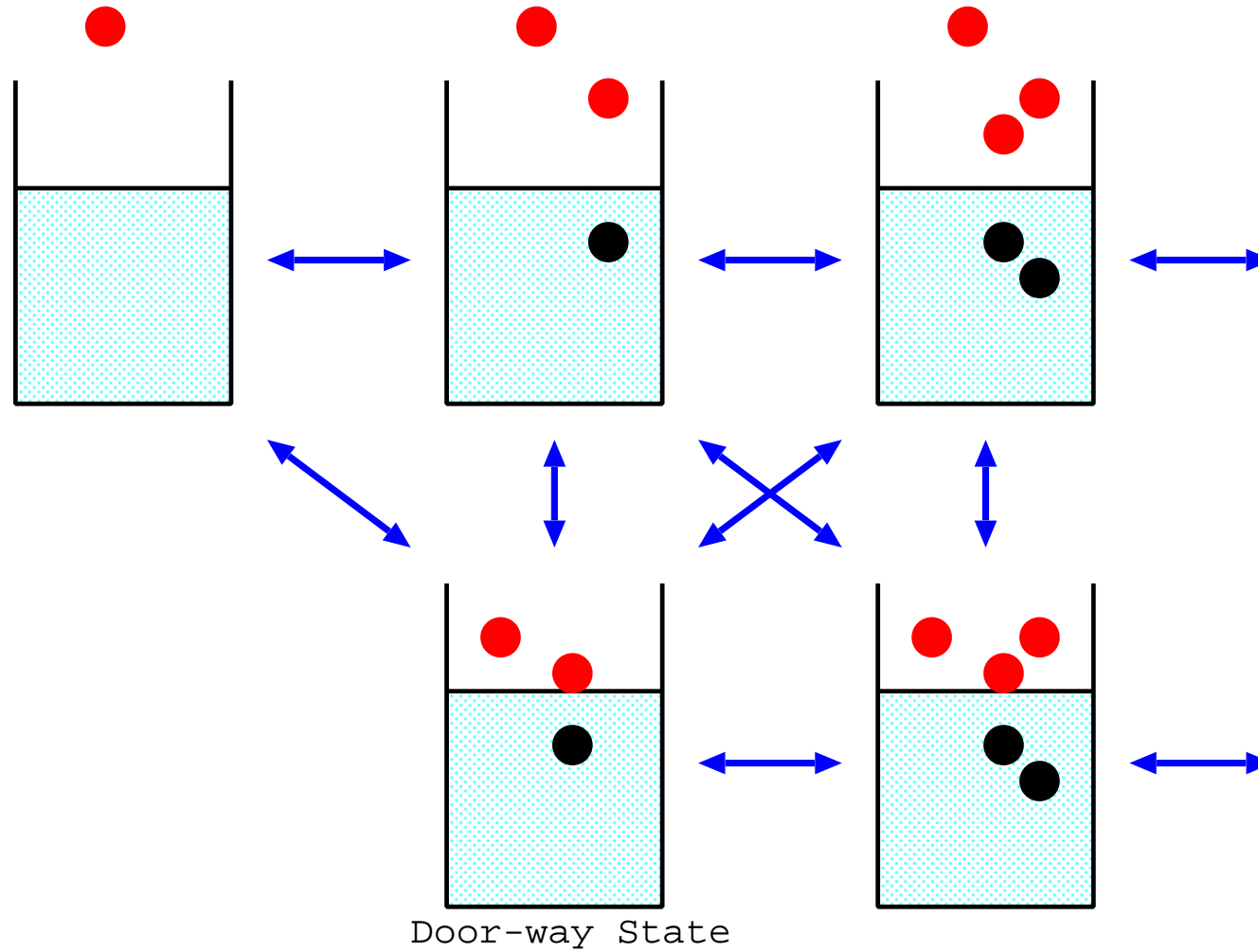
- Exciton Model
 - Nuclear state — n -particle ($n - 1$)-hole state
 - Transition rate — $\lambda_{nn'} = \frac{2\pi}{\hbar^2} |M|^2 \rho_{n'}$
 - Solve a master equation — time-dependent occupation probability $P(n, t)$ of the n -exciton states.
 - The matrix element $|M|^2$ is regarded as an adjustable parameter.
 - Generally the exciton model gives **a good fit to the energy distribution of emitted particles**, however, a traditional exciton model cannot calculate angular distribution.
- Intra-Nuclear Cascade (INC)
 - Cannot apply to low-energy reactions.

■ | Pre-Equilibrium Theories (II)

Quantum Mechanical Theory

- Feshbach, Kerman, and Koonin (1980) — FKK
- An extension of DWBA to the continuum state
- Particle-Hole excitation — similar to the exciton model
- Q -space (Multistep Compound, MSC)
 - Final state is bound
 - Residual System: $2p-1h$, $3p-2h$, $4p-3h$, ...
 - MSC gives an isotropic angular distribution
- \mathcal{P} -space (Multistep Direct, MSD)
 - At least one particle is unbound
 - Residual System: $1p-1h$, $2p-2h$, $3p-3h$, ...
 - MSD has a forward-peaked angular distribution

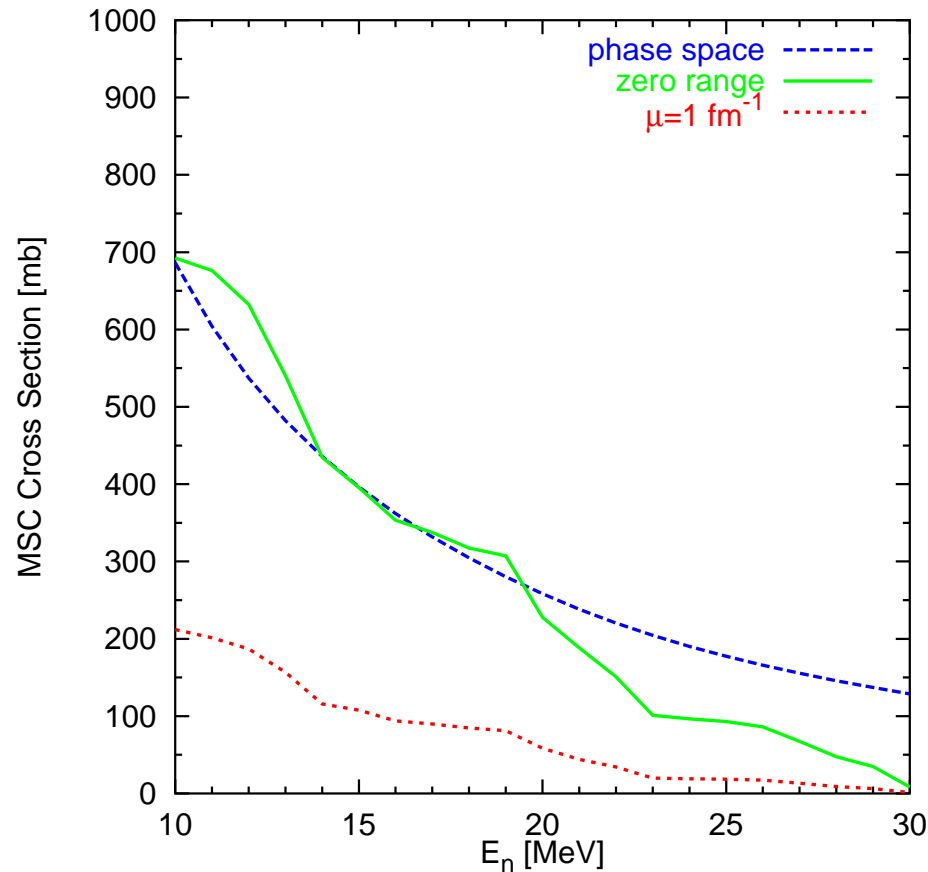
\mathcal{P} -Space, \mathcal{Q} -Space





Multistep Compound Process (MSC)

Strength of $2p$ - $1h$ Formation

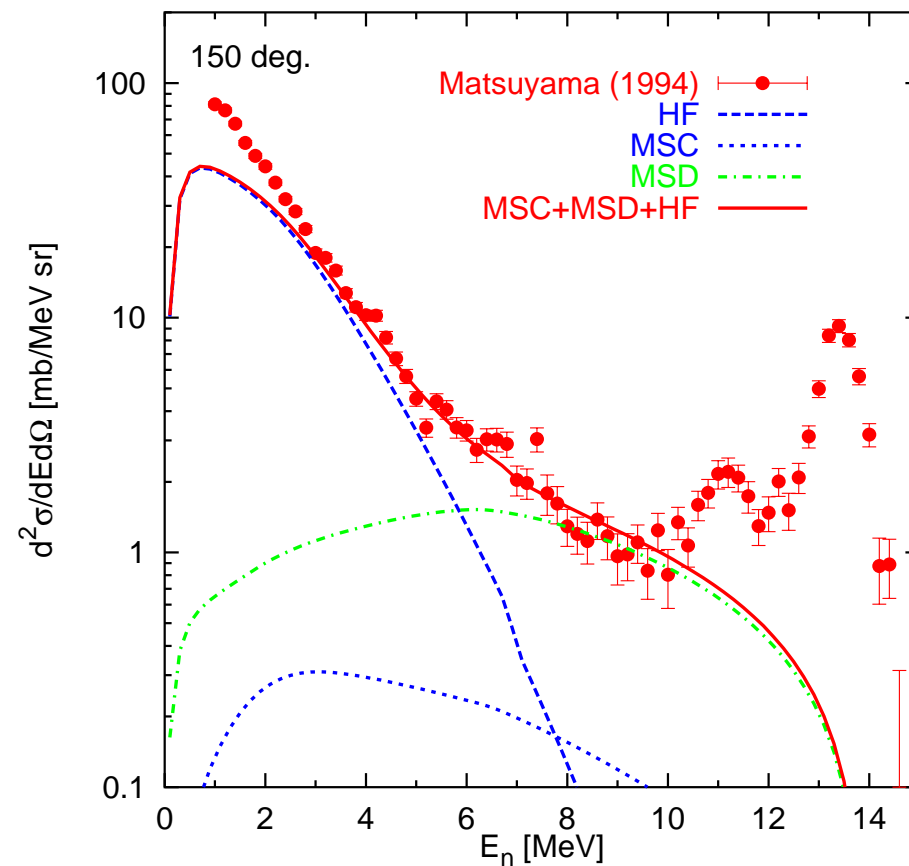
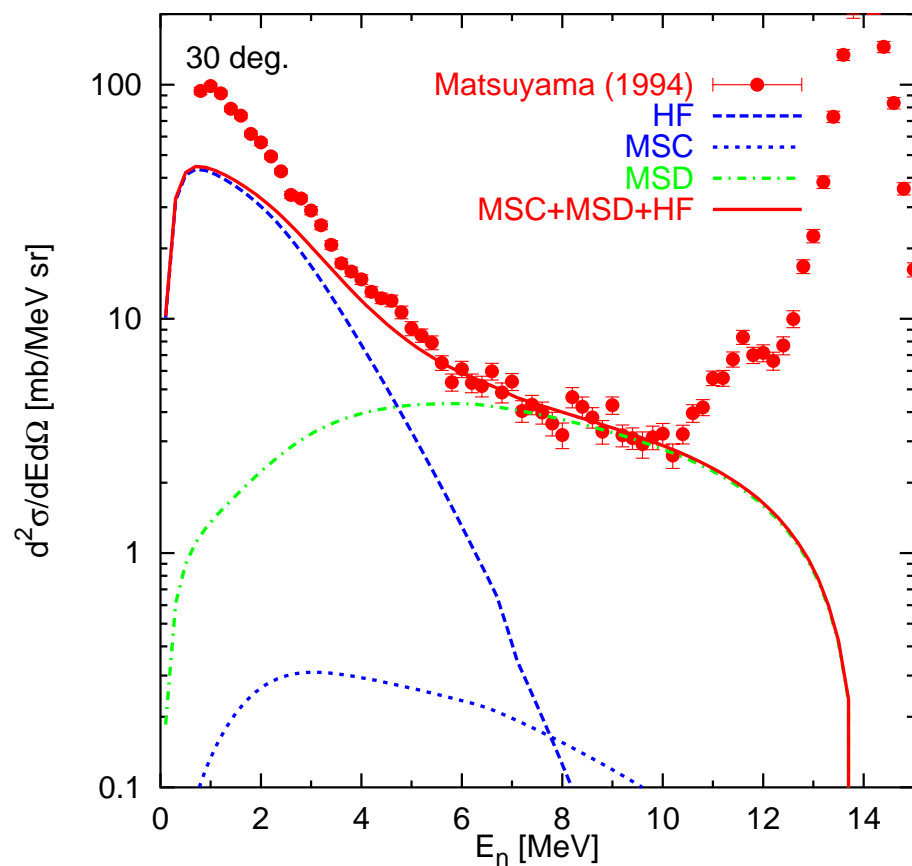


- $^{93}\text{Nb} + n$ reactions.
- Microscopic calculation of doorway state formation cross section.
 - DWBA transition matrix element \times final state density.
 - $\langle p'(ph^{-1}) | \mathcal{V} | 0\chi_a \rangle \rho_{2p1h}$
- Phase-space approximation of Chadwick and Young.
 - Estimated a fraction of MSC by using a final state density.



Strength of MSC

DDX Data of ^{93}Nb at 14 MeV



MSD Theories

- **FKK**: Feshbach, Kerman, Koonin (1980)
 - On-Shell Approximation for Green's Function
- **TUL**: Tamura, Udagawa, Lenske (1982)
 - Random Phase Approximation (RPA)
 - Adiabatic Approximation for the Second Step
- **NWY**: Nishioka, Weidenmüller, Yoshida (1988)
 - Random Matrix Theory (GOE)
 - Sudden Approximation for the Second Step
- **SCDW**: Luo, Kawai, Weidenmüller (1991,1992)
 - Eikonal Approximation for the Second Step

One-step process is dominant below 20 MeV, and the one-step expression of FKK, TUL, and NWY is the same (in principle).

Comparison of FKK, TUL, and NWW

	FKK	TUL	NWW
Approximation	on-shell	Adiabatic	Sudden
Statistical Average	Each	Each	Final
State density	$\rho_{1p1h} \otimes \rho_{1p1h}$	$\rho_{1p1h} \otimes \rho_{1p1h}$	ρ_{2p2h}
Model	Equidistant	RPA	GOE
Interference	No	No	Yes

Time Scale

TUL *Adiabatic Approximation*

1p-1h 2p-2h

NWW *Sudden Approximation*

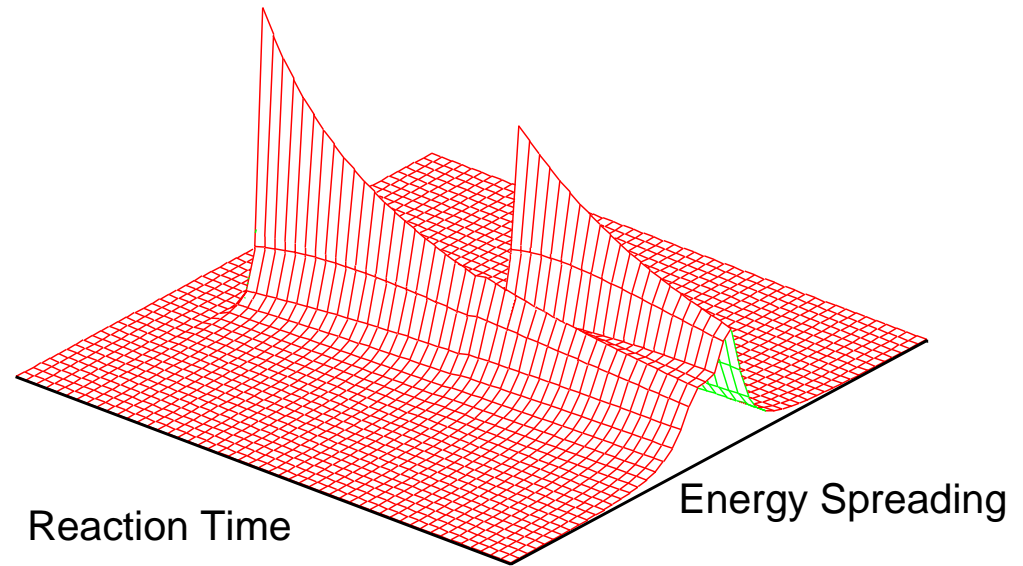
1p-1h 2p-2h 3p-3h

NWY Theory

- Quantum Mechanical Theory for the Pre-Equilibrium Process (MSD)
- Residual System Statistics
- use so-called **True Level Density**, $\rho_{\mu m}(E)$ which includes a **Residual Interaction**, V
- The Level (State) Density is different from the equidistant-spacing model of Ericson and Williams.
- one-step: (DWBA) \times ($1p$ - $1h$ Strength)
- two-step: (2nd order DWBA) \times ($2p$ - $2h$ Strength)
- **Sudden Approximation** is made for the second step.

Sudden Approximation

- An additional p - h pair creation is much faster than residual configuration mixing.
- An intermediate state is always an $1p$ - $1h$ state.
- Amplitudes for the different paths to reach the same final state interfere each other.



Residual System Statistics

- Observable Cross Sections
 - various microscopic 2p-2h state excitation averaged over the residual state
- True Level Density of Sato, Takahashi, and Yoshida
Based on the Random Matrix Model
Z. Phys. A, **339**, 129 (1991).
- Hamiltonian for the Nuclear System $H = h + V$
 - h independent particle model
$$(h - \epsilon_{m\mu})|m\mu\rangle = 0$$
 - V residual interaction: V is assumed to form a GOE which is characterized by a second moment \mathcal{M}_{mn}

True Level Density

Unperturbed State Density

$$\rho_m^{(0)J\pi}(E) = \sum_{\mu} \delta(E - \epsilon_{m\mu})$$

Exciton State Density for fixed $J\pi$

$$\rho_m^{J\pi}(E) = - \sum_{\mu} \frac{1}{\pi} \text{Im} \frac{1}{E - \epsilon_{m\mu} - \sigma_m^{J\pi}(E)}$$

Saddle Point Equation

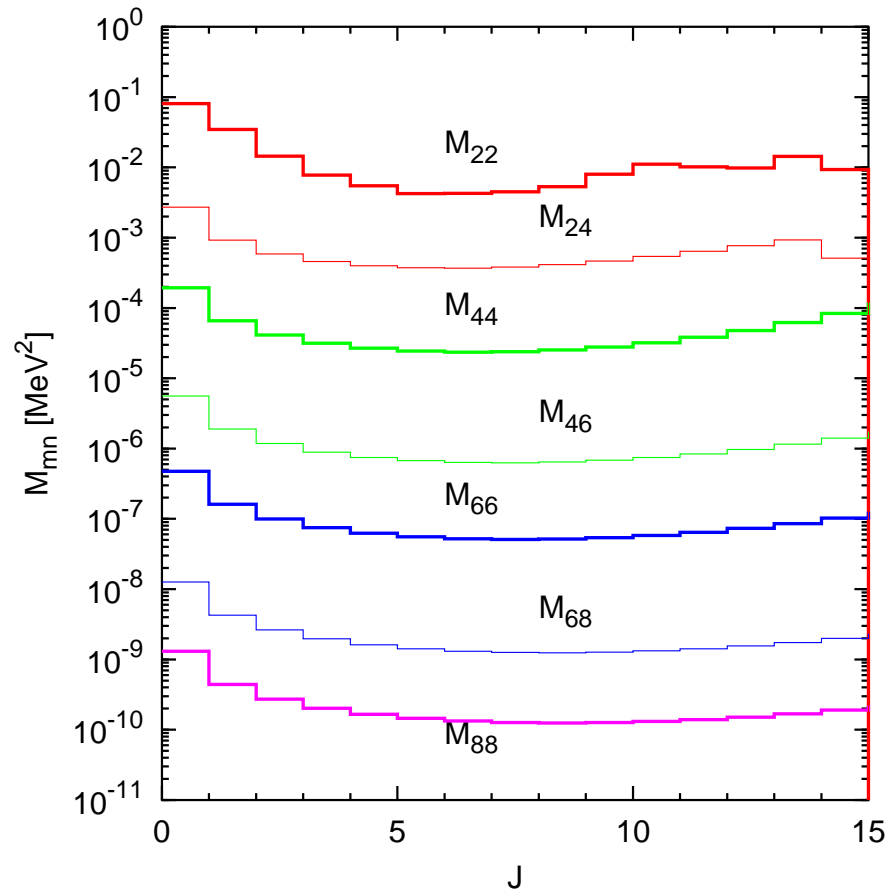
$$\sigma_m^{J\pi}(E) = \sum_n \mathcal{M}_{mn} \int \rho_n^{(0)J\pi}(\epsilon) \frac{1}{E - \epsilon - \sigma_n^{J\pi}(E)} d\epsilon$$

Second Moment

Calculated for ^{208}Pb , with the M3Y-Paris Interaction which contains the central part V^C and the tensor part V^T

$$V^C = \sum V_{st}^C (\sigma_1 \cdot \sigma_2) (\tau_1 \cdot \tau_2) Y(\mu r)$$

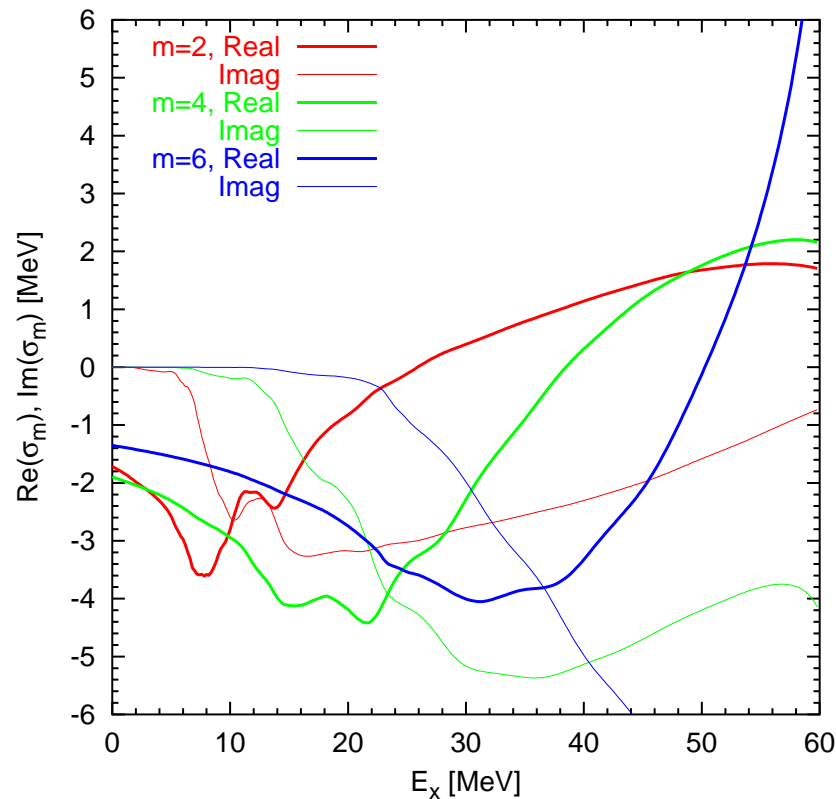
$$V^T = \sum V_t^T (\tau_1 \cdot \tau_2) S_{ij} Y(\mu r)$$



Saddle Point Value

true level density

$$\rho_m^{I_B}(E_x) = -\frac{1}{\pi} \text{Im} \frac{1}{E_x - \epsilon_B - \sigma_m^{I_B}(E_x)}$$

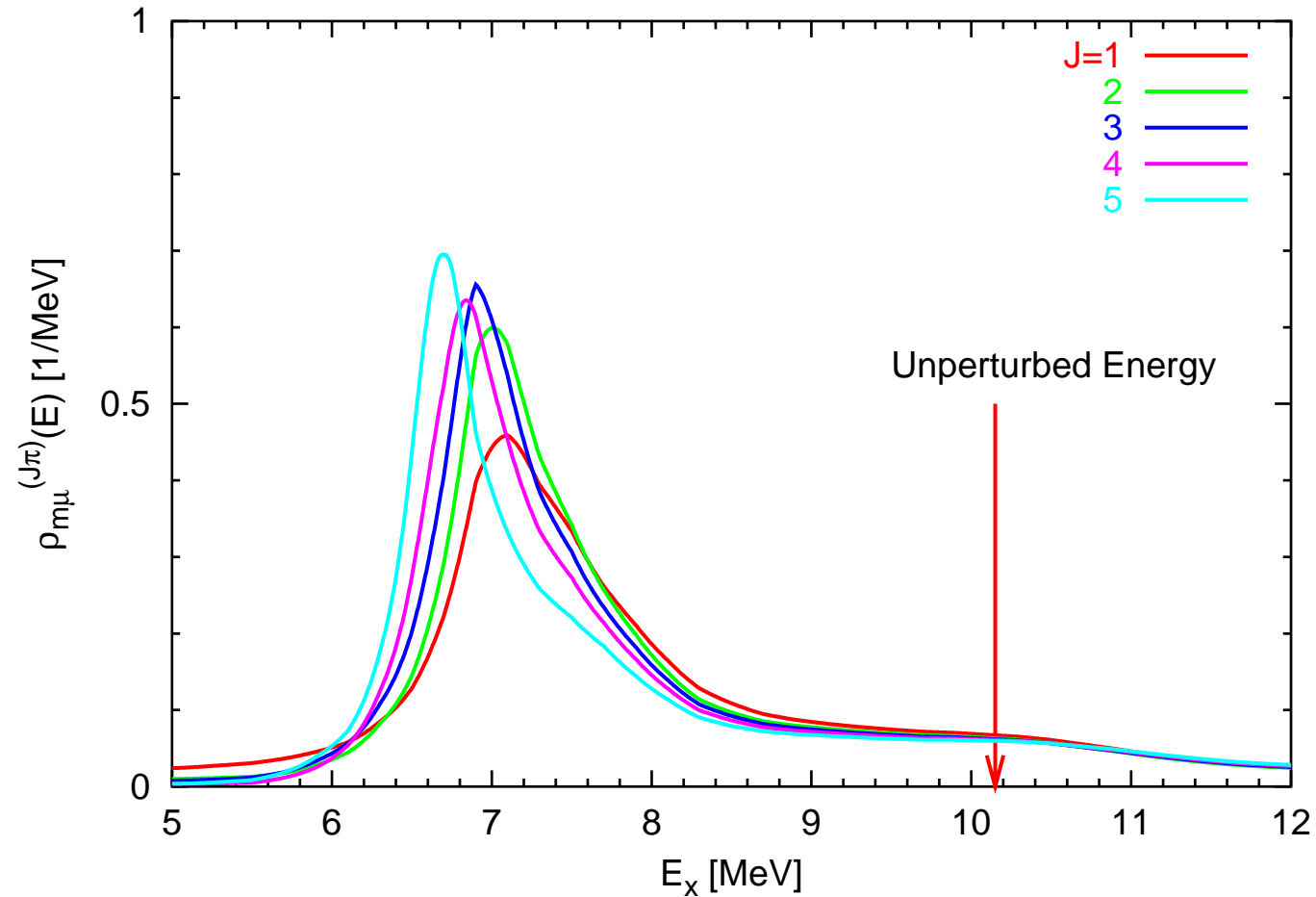


- $\text{Re}(\sigma_m)$ = energy shift
- $\text{Im}(\sigma_m)$ = energy spread

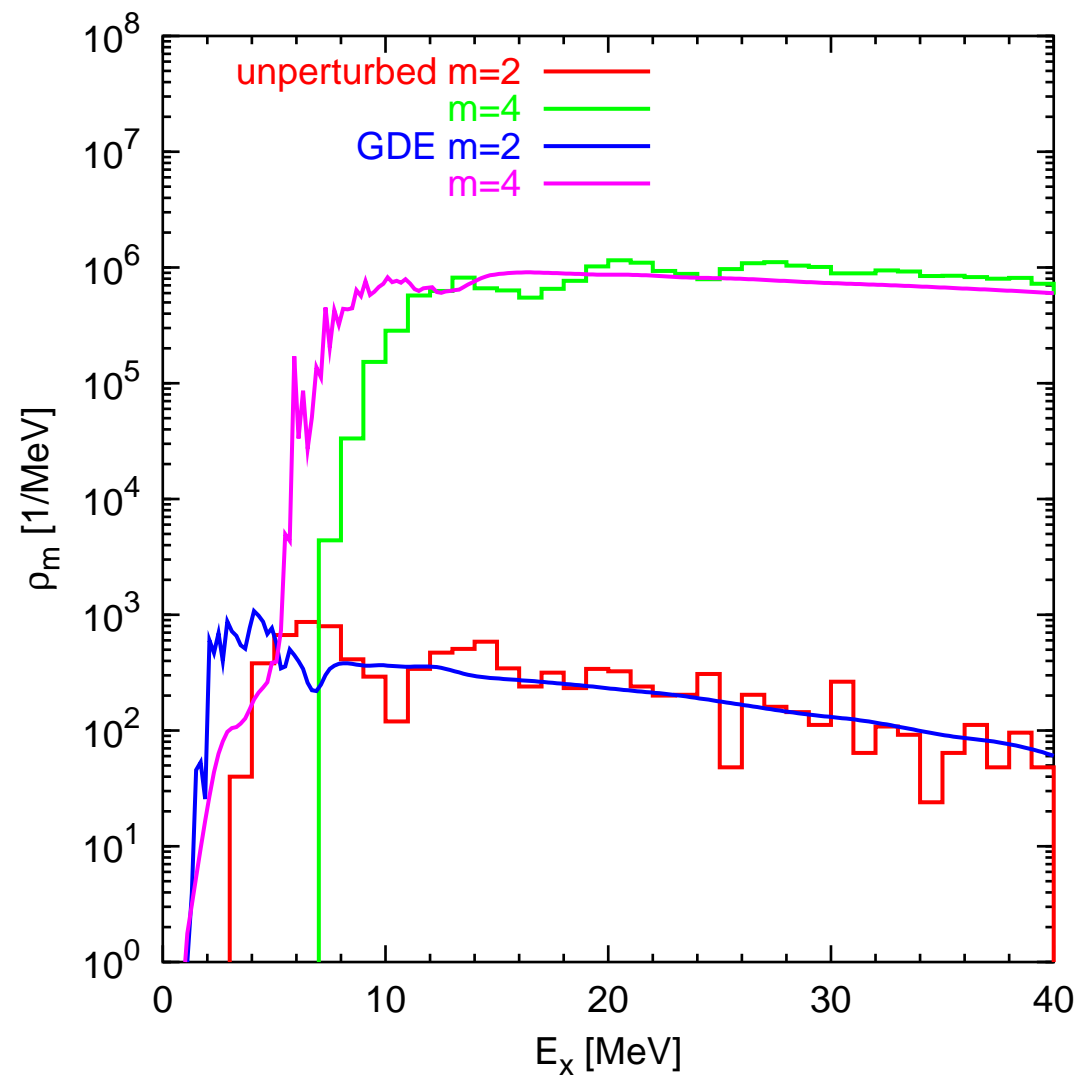


Example: 1p-1h State Distribution

^{208}Pb , $|1g_{7/2}(0h_{9/2})^{-1}\rangle_\nu$, $E_x = 10.15 \text{ MeV}$



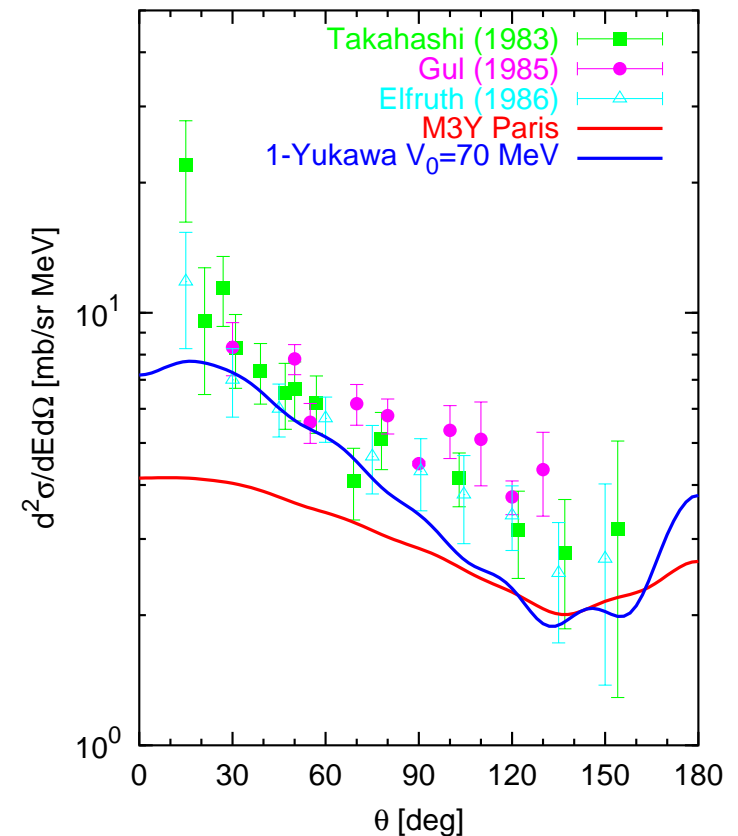
p-h State Density



One-Step Cross Section

$$\frac{d^2\sigma_{ba}}{dEd\Omega} = \frac{(2\pi)^4}{k_a^2} \sum_{\mu} |\langle \chi_b^{(-)} u_{m\mu} | \mathcal{V} | \chi_a^{(+)} u_0 \rangle|^2 \rho_{m\mu}(E_x)$$

- $^{208}\text{Pb}(n, n')$ reaction at:
 $E_{in} = 14.5$,
 $E_{out} = 7.5$ MeV
- Walter-Guss' optical potential
- M3Y interaction
- Yukawa interaction:
 $V_0 = 70$ MeV, and $r_0 = 1$ fm



Bonetti Approach

FKK One-step Calculation

$$\begin{aligned}\frac{d^2\sigma_{ba}}{dEd\Omega} &= \frac{(2\pi)^4}{k_a^2} \sum_{\mu} |\langle \chi_b^{(-)} u_{m\mu} | \mathcal{V} | \chi_a^{(+)} u_0 \rangle|^2 \rho_{m\mu}(E_x) \\ &= \sum_j \frac{(2\pi)^4}{k_a^2} |\langle \chi_b^{(-)} u_{m\mu} | \mathcal{V} | \chi_a^{(+)} u_0 \rangle|^2 (2j+1) \hat{\rho}_{1p1h}(E_x, j) \\ &= \sum_j \left\langle \left(\frac{d\sigma_{ba}}{d\Omega} \right)_{DWBA} \right\rangle_j \hat{\rho}_{1p1h}(E_x, j)\end{aligned}$$

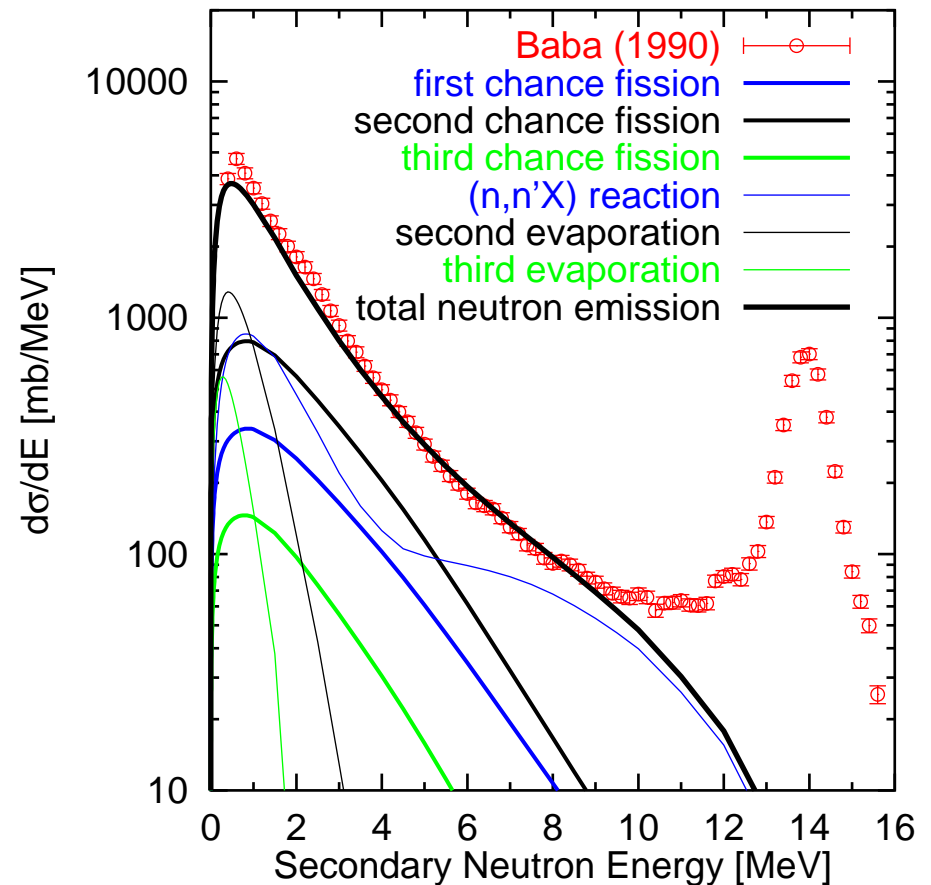
- Averaged DWBA cross section
 - particle-hole excitation, with angular momentum transfer of j
- Phenomenological level density $\hat{\rho}_{1p1h}(E_x, j)$

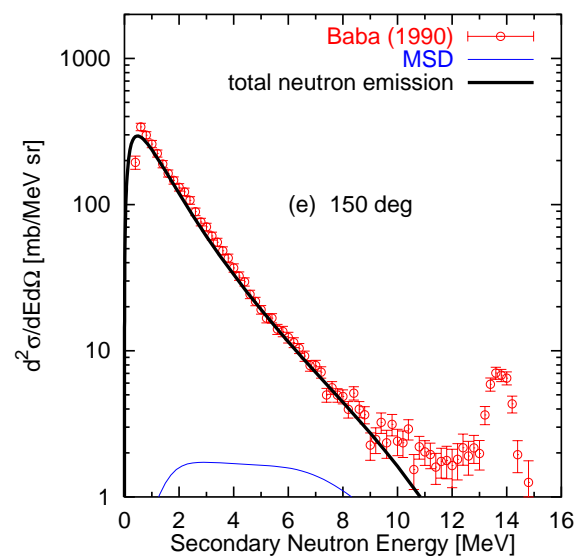
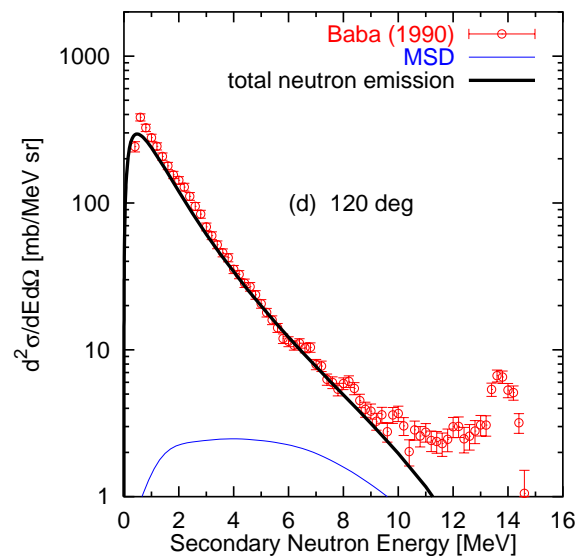
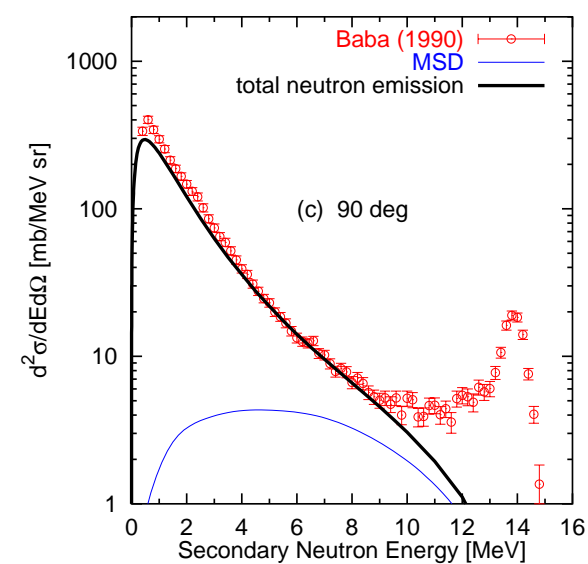
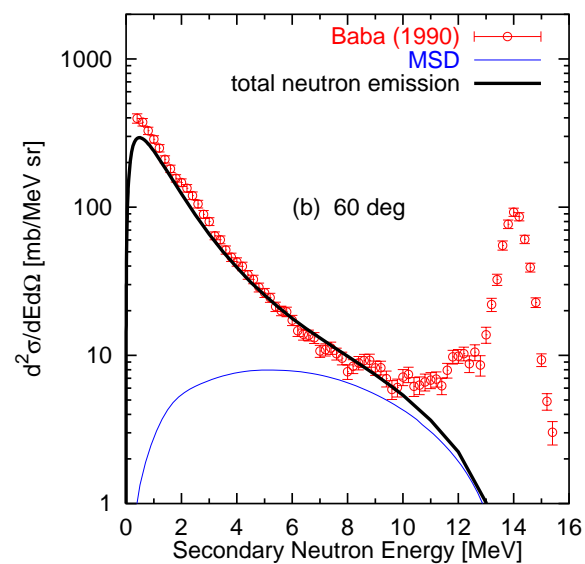
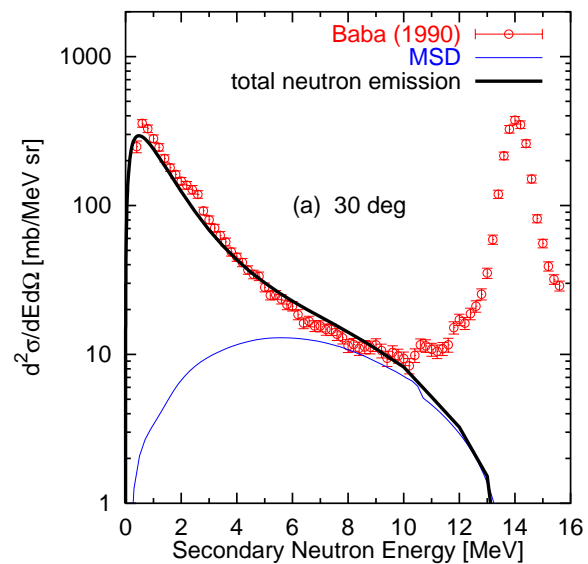
Example — U-238

Neutron Inelastic Scattering

$$^{238}\text{U}(n, xn), E_n = 14.1 \text{ MeV}$$

- DWBA formfactor for p - h excitation is a simple Yukawa-form.
- The strength of Yukawa interaction V_0 is adjusted to experimental DDX data.
- $V_0 = 50.5 \text{ MeV}$
- Multi-Step Compound (MSC) included.



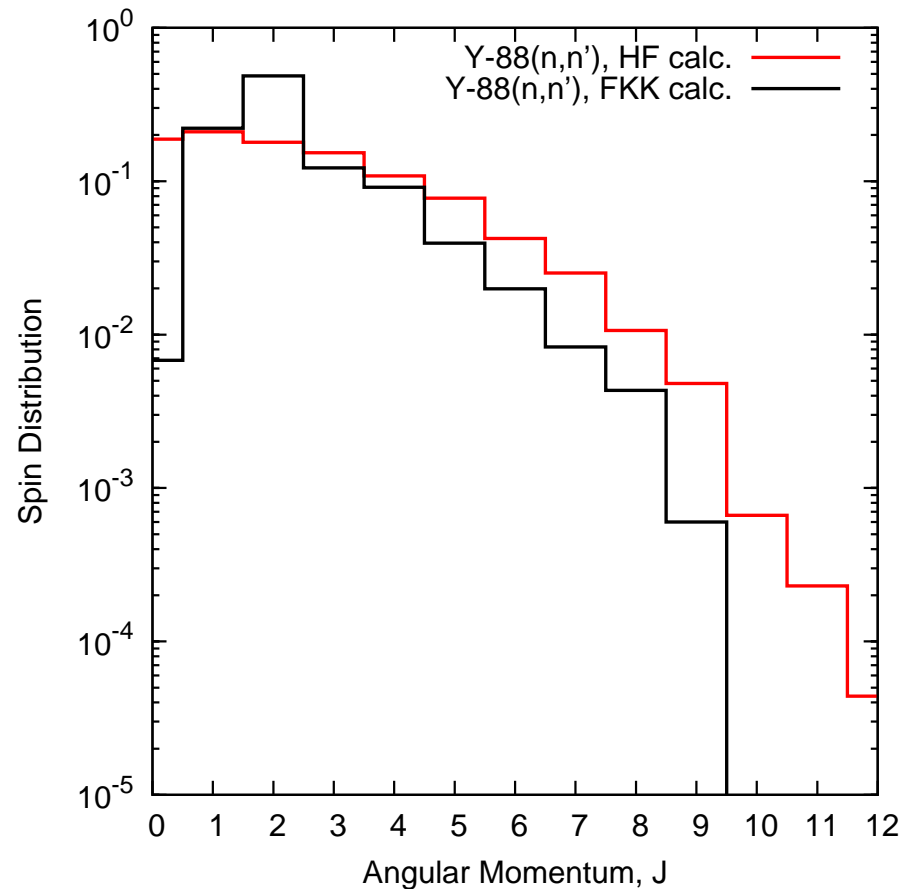




Pre-Compound Spin Distribution

Spin Distribution of Residual Nucleus

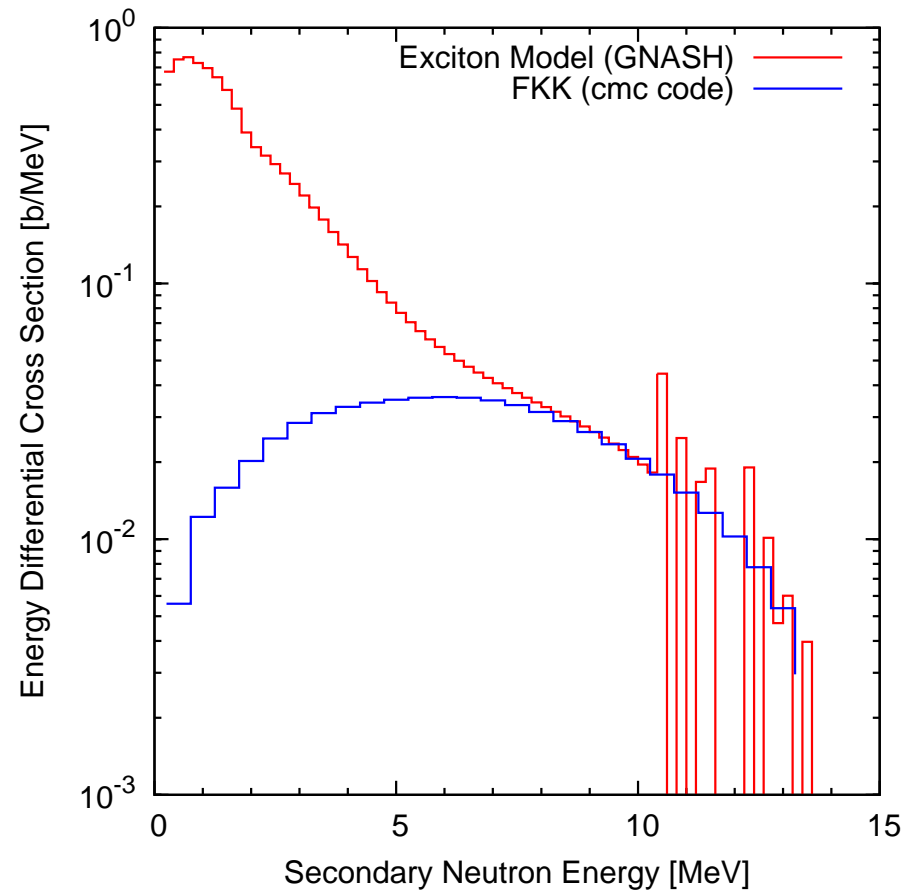
- $^{87}\text{Y} + n$ reaction at $E_n = 14$ MeV.
- Excitation energy of 6 MeV
- Koning-Delaroche global optical potential for incident neutron.
- Ground state spin $I = 1/2^-$ is assumed to be zero.
- FKK calculation does not have a high J component, because of $p-h$ configurations.



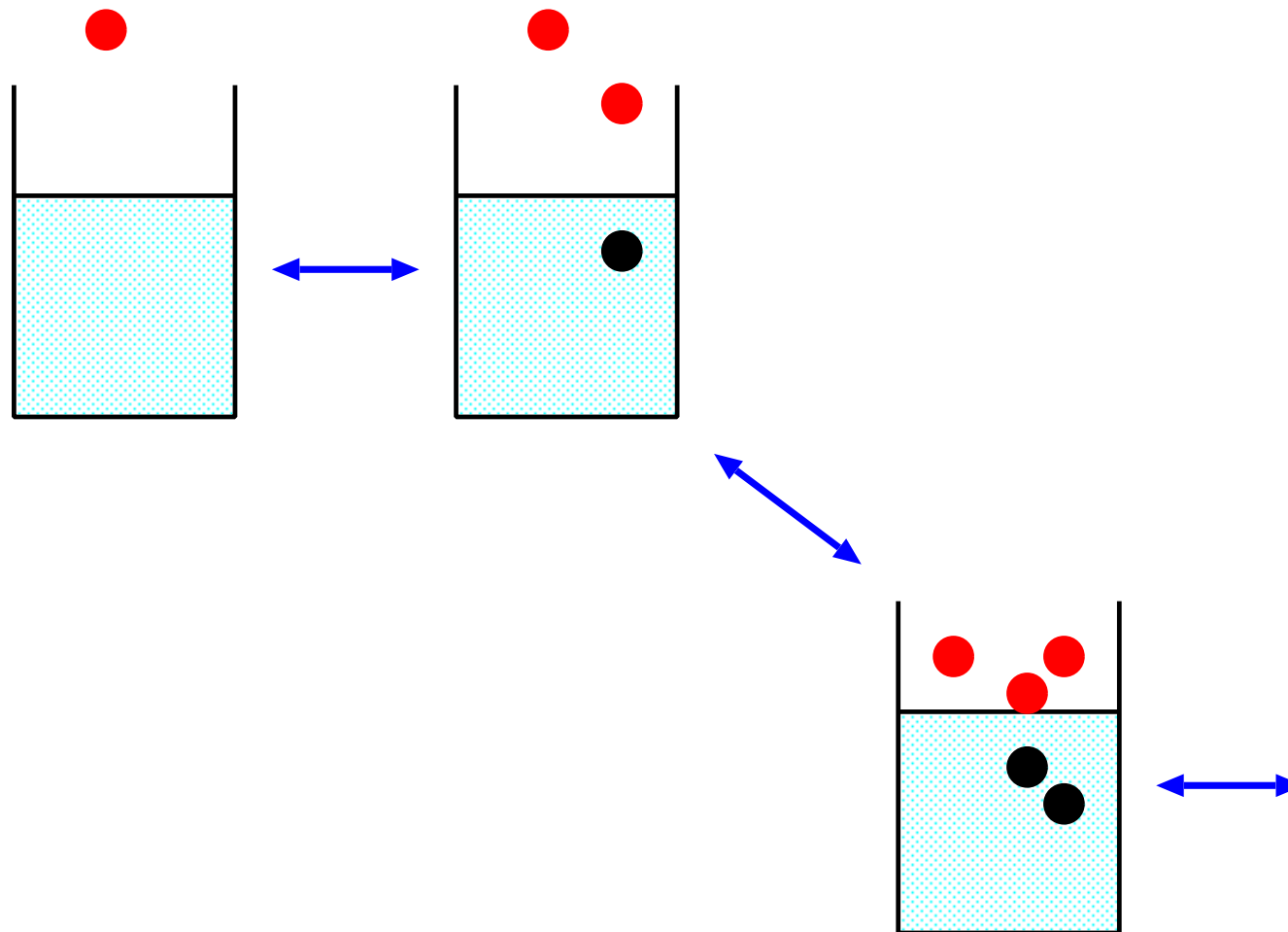
Energy Spectrum

Comparison of FKK calc. with Exciton Model

- 14 MeV neutron induced, neutron emission reaction on ^{87}Y .
- The FKK MSD calculation is re-normalized to GNASH calculation (Exciton model).
- The MSD cross section is 20% of total reaction cross section.



Dominant Process at 14 MeV



Concluding Remarks

- Spin-distribution of the pre-equilibrium process can be calculated with quantum mechanical theories — FKK, NWY, and TUL.
- Microscopic calculations of both MSC and MSD were described.
- MSC contribution is very small.
- For $^{87}\text{Y}(n, n')$ reaction at 14 MeV, MSD is about 20% of total reaction cross section, and the compound reaction is still a dominant process at low energies.
- Spin-distribution of the residual nucleus may not have a big impact on the surrogate reaction technique if an incident neutron energy is not so high, however, further study is needed (quantitatively):
 - Probabilities of γ -ray cascade
 - High energy reactions
 - Large target spin